in developing the entrainment equation from Equation (6) is applicable to other rotary cylinders. Equation (9) or Equation (10) is useful in investigating the operation of a rotary petroleum coke calciner.

ACKNOWLEDGMENT

The author wishes to thank Drs. J. R. Friday and F. E. Sage for reviewing this article and Mr. G. A. Sweany for discussion on a portion of the study. Permission to publish this article given by Continental Oil Company is gratefully acknowledged.

NOTATION

a = constant
C_g = concentration of fines in the gas, kg/m^3
C_f = concentration of entrainable fines in solid, wt. %
d = density of solid, kg/m^3
D = inside diameter of cylinder, m
D_f = diameter of the largest fine particle which can be entrained, mm
D_s = diameter of feed solid particles, m
F = feed rate of solid, kg/s
F_1 = $D/d/(\pi D^2/4)$, m^3/s/m^2
G = gas flow rate, kg/s
G_1 = $G/(\pi D^2/4)$, kg/s/m^2
K, K_1, K_2, K_3 = proportionality constants
L = length of cylinder, m
m = weight fraction of feed particles with size larger than D_s
n = solid particle size distribution parameter
N = cylinder rotation speed, rad/s
Re = $DU\rho/\mu$
S = cylinder slope, m/m
T = average of gas exit temperature T_e and maximum bed temperature T_m, °K
U_t = average fines terminal velocity, m/s
U = gas velocity in the direction of cylinder axis, m/s
V = bed holdup volume per unit cylinder length, m^3/m
W = fines entrainment rate, kg/s
X = fraction of cylinder volume occupied by solid bed
Z = $W/(NFD)^{1/2}/T^{3/4}$
θ = dynamic angle of repose of solid, rad
ρ = density of gas, kg/m^3
μ = viscosity of gas, N · s/m^2

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Manuscript received April 22 and accepted April 24, 1974.

Optimal Performance of Equilibrium Parametric Pumps

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In previous papers (Chen et al., 1971, 1972, 1973), continuous and semicontinuous pumps have been analyzed in terms of an equilibrium theory of pump performance. Mathematical expressions for the concentration transients have been given for the region of infinite separation factors and two regions of finite separation factors. In addition, it has been shown that when the penetration distance of the cold cycle is less than or equal to that of the hot cycle and the height of the column, the pump with feed at the enriched end is capable, as a theoretical limit, of complete removal of solute from one product stream and, at the same time, gives arbitrarily large enrichment of solute in the other product stream. In this note we extend the mathematical models previously developed to determine the optimal pump performance. The model system used is sodium nitrate-water on an ion retardation resin adsorbent. Emphasis is given on the operating conditions necessary to achieve high separation factors with the maximum yield. Information in connection with such optimal values of operating parameters is essential for design or scale-up purposes.

THEORY

Let us assume that a dilute solution includes s adsorbable components in an inert solvent. Let us assume further that the solution may be treated as s pair of pseudo binary systems. Each system includes one solute and the common inert solvent, each with a dimensionless equilibrium parameter b_i and the corresponding values of L_{1i} and L_{2i}. Furthermore

$$b_1 > b_2 \ldots\ldots b_k \geqq \phi_B > b_{k+1} \ldots\ldots > b_s \qquad (1)$$

for the continuous pump, and

$$\left(\frac{2b}{1-b}\right)_1 > \left(\frac{2b}{1-b}\right)_2 \ldots\ldots\ldots \left(\frac{2b}{1-b}\right)_k \geqq \phi_B > \left(\frac{2b}{1-b}\right)_{k+1} \ldots\ldots > \left(\frac{2b}{1-b}\right)_s \qquad (2)$$

for the semicontinuous pump. Also

$$L_{2i} = \frac{v_0\,(1+\phi_B)}{(1+b_i)(1+m_0)}\left(\frac{\pi}{\omega}\right) \leqq h \qquad (3)$$

where $i = 1, 2, \ldots.\ k$.

At steady state ($n \rightarrow \infty$) the components $i = 1, 2, \ldots$ k for which operations occur in region 1 would appear only in the top product stream, and the remaining components ($k + 1, \ldots.\ s$) would appear in both top and bottom

TABLE 1. EXPERIMENTAL AND MODEL PARAMETERS

$\frac{\pi}{\omega} = 1{,}200$ s., $T_1 = 338°$K, $T_2 = 278°$K, $b = 0.04$, $m_0 = 0.59$

$h = 0.9$ m, Feed* $= 0.4167 \times 10^{-2}$ cm³/s, $y_0 = 0.25 \times 10^{-4}$ g moles/cm³

		$P_{B\max}\left(\frac{\pi}{\omega}\right)$ cm³	$Q_{\max}\left(\frac{\pi}{\omega}\right)$ cm³	$P_B\left(\frac{\pi}{\omega}\right)$ cm³	$Q\left(\frac{\pi}{\omega}\right)$ cm³	C_1**	C_2***	L_1 m	L_2 m	Region
1.	Semicontinuous	2.88	34.56	1.0	20	0.215	0.215	0.52	0.50	1
2.	Semicontinuous	2.88	34.56	1.0	30	0.167	0.143	0.78	0.74	1
3.	Semicontinuous	2.88	34.56	2.0	30	0.167	0.143	0.78	0.77	1
4.	Semicontinuous	2.88	34.56	4.0	30	0.167	0.143	0.78	0.82	2
5.	Semicontinuous	2.88	34.56	1.8	40	0.125	0.125	1.04	1.01	3
6.	Continuous	1.44	36.00	1.0	30	0.167	0.143	0.76	0.75	1
7.	Continuous	1.44	36.00	2.0	30	0.167	0.143	0.73	0.77	2

* Except Run 5 for which Feed $= 0.8334 \times 10^{-2}$ cm³/s.
** C_1 = top reservoir dead volume/displacement.
*** C_2 = bottom reservoir dead volume/displacement.

product streams. In the extreme case where $k = s$ the bottom product would consist only of pure solvent and the top product must contain all of the solutes supplied by the feed stream (Chen et al., 1974).

The optimal pump performance would be interpreted as the achievement of separation with maximum production of bottom product and complete removal of solutes 1, 2, ... k. Thus, one can see from Equations (1) to (3) that this would occur when

1. $L_{2k} = h$ (that is, the pump is operated just on the verge of breakthrough of solute k from the top to the bottom of the column), and
2. $\phi_B = b_k$ for the continuous pump.

$$\phi_B = \left(\frac{2b}{1-b}\right)_k \text{ for the semicontinuous pump.}$$

Under these circumstances it can be shown that the maximum (or optimal) reservoir displacement flow rate is

$$Q_{\max.} = \begin{cases} A\,h\,\epsilon(1+m_{0k}) \Big/ \left(\frac{\pi}{\omega}\right) & \text{for the continuous pump} \\ A\,h\,\epsilon(1+m_{0k})(1-b_k) \Big/ \left(\frac{\pi}{\omega}\right) & \text{for the semicontinuous pump} \end{cases} \quad (4)$$

and the maximum bottom product flow rate is

$$\mathbf{P_{B\ max.}} = \begin{cases} Q_{\max.}\,b_k = A\,h\,\epsilon(1+m_{0k})b_k \Big/ \left(\frac{\pi}{\omega}\right) & \text{for the continuous pump} \\ Q_{\max.}\left(\frac{2b}{1-b}\right)_k = A\,h\,\epsilon(1+m_{0k})2b_k \Big/ \left(\frac{\pi}{\omega}\right) & \text{for the semicontinuous pump} \end{cases} \quad \mathbf{(5)}$$

The continuous pump is characterized by a steady flow for both feed and product streams during both upflow and downflow cycles, whereas the semicontinuous pump is operated batchwise during upflow and continuous during downflow. The quantity b_k in Equations (4) and (5) is a measure of the extent of movement of solute k between phases as a result of a change in column temperature. b_k may be as small as zero for a system in which the equilibrium distribution is insensitive to temperature or as large as unity for which distribution is highly temperature sensitive.

RESULTS AND DISCUSSION

The optimal pump performance was studied for the case of $NaNO_3$ separation from H_2O using an ion retardation resin (Bio-Rad AG 11 A8) as adsorbent. The experimental apparatus is identical to that used previously (Chen et al., 1972). Two modes of pump operation were considered—continuous and semicontinuous. The experimental parameters are shown in Table 1, and the data are plotted in Figures 1 and 2. The equations previously derived (Chen et al., 1972, 1973) were used to calculate the concentration transients, and computed results corresponding to the experimental runs are also presented in Figures 1 and 2. These results compare reasonably well with the observed values. The equilibrium parameters b_i and m_0 used in the computations were obtained by a method described by Chen et al. (1972) and found to be 0.04 and 0.59, respectively. Note that they are a function of hot and cold temperatures only.

Figure 1 shows the effects of Q and P_B on the product concentrations for the semicontinuous pump. As long as $Q \le Q_{\max}$ $\left(\text{or } Q\frac{\pi}{\omega} \le Q_{\max}\frac{\pi}{\omega}\right)$ and $P_B \le P_{B\max}$ $\left(\text{or } P_B\frac{\pi}{\omega} \le P_{B\max}\frac{\pi}{\omega}\right)$, $\langle y_{BP2}\rangle_n/y_0$ decreases as n increases

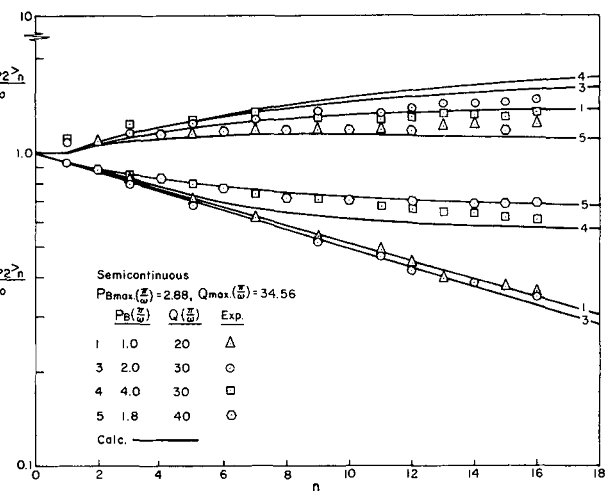

Fig. 1. Effects of P_B and Q on product concentrations.

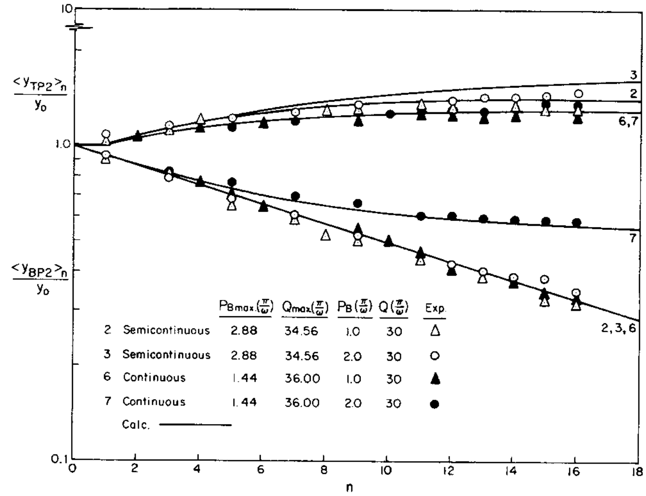

Fig. 2. Comparison of semicontinuous and continuous parametric pumps.

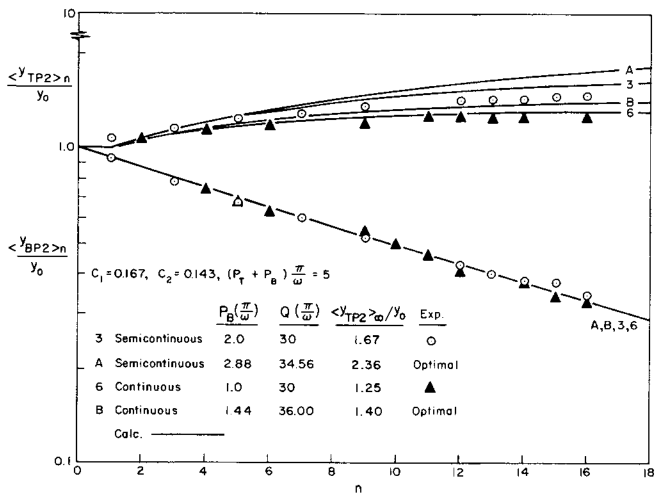

Fig. 3. Comparison of concentration transients for $P_B < P_{B\,\max}$. and $Q < Q_{\max}$. with the optimal theoretical results.

and will approach zero as theory predicts (curves 1 and 3). The slope (of log $(\langle y_{BP2} \rangle_n / y_0)$ vs. n) depends on the values of C_2 and b, where C_2 is defined as the ratio of dead volume of the bottom reservoir to the displacement. If, in a pump originally operated in a region where $Q < Q_{\max}$ and $P_B < P_{B\,\max}$, P_B is increased until it exceeds $P_{B\,\max}$ or Q becomes greater than $Q_{\max}$, the steady state behavior of the pump abruptly switches from a mode in which solute is completely removed from the lower reservoir to one in which solute removal is incomplete (curves 4 and 5). Hence, $Q_{\max}$ and $P_{B\,\max}$ are the operating conditions necessary to accomplish separation with the maximum production of bottom product and infinite separation factors. It should be pointed out that the crossing of the bounadry $Q = Q_{\max}$ or $P_B = P_{B\,\max}$ may also be thought of as switching from Region 1 to 3 or 1 to 2. A more detailed explanation concerning regions of pump operation is given by Chen and Hill (1971).

As shown in Figure 2 the performance characteristics of both continuous and semicontinuous pumps are similar in nature. If the pumps are operated with $Q \leqq Q_{\max}$ and $P_B \leqq P_{B\,\max}$ (curves 2, 3, and 6), the bottom product concentration decreases as n increases and the separation factor becomes large as n becomes large. The principal difference between the two pumps is the difference in $P_{B\,\max}$ and $Q_{\max}$ [Equations (4) and (5)]. For the continuous pump $P_{B\,\max}$ is half of that for the semicontinuous pump, and $Q_{\max}$ is greater than that for the semicontinuous pump by a factor of $1/(1 - b_k)$. For semicontinuous operation the average flow rate during a complete cycle is $P_B/2$. Thus, in terms of the average value both semicontinuous and continuous pumps have the same maximum bottom product flow rate. However, for large b_k, $Q_{\max}$ in the semicontinuous pump may become quite small relative to that in the continuous pumps, that is, one can have a much smaller reservoir volume by using semicontinuous operation.

Figure 3 shows a comparison of experimental and calculated concentration transients for $P_B < P_{B\,\max}$ and $Q < Q_{\max}$ with the optimal theoretical results. One can see that variations in P_B, Q, and the mode of pump operation have no effect on the bottom product concentration $\langle y_{BP2} \rangle_n / y_0$. In every case, at the steady state $(n \to \infty)$, solute removal from the bottom product stream can be complete, and the top product stream must carry away all of the solute supplied by the feed stream. However, the pumps with $P_B = P_{B\,\max}$ and $Q = Q_{\max}$ (or the optimal operating conditions) are ideal separation devices in the sense that they can continuously or semicontinuously separate a system into one fraction completely free of solute and another fraction enriched the solute to the maximum degree.

NOTATION

A = column cross-sectional area, m^2
b = equilibrium parameter, dimensionless
h = column height, m
L = penetration distance, m
m_0 = equilibrium parameter, dimensionless
n = number of cycles of pump operation
P_B = bottom product volumetric flow rate, cm^3/s
Q = reservoir displacement rate, cm^3/s
v_0 = interstitial velocity based on the reservoir displacement rate, m/s
y = concentration of solute in the liquid phase, g moles/cm^3
ϵ = void fraction in packing, dimensionless
$< >$ = average value
ϕ_B = P_B/Q
$\frac{\pi}{\omega}$ = duration of half cycle, s

Subscripts

0 = initial condition
1 = upflow
2 = downflow
BP = bottom product
i = solute i
max = optimal or maximum condition
TP = top product

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Manuscript received April 4, 1974; revision received and accepted June 7, 1974.